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BMN operators with vector impurities, $\mathbb{Z}_2$ symmetry and pp-waves

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Abstract: We calculate the coefficients of three-point functions of BMN operators with two vector impurities. We find that these coefficients can be obtained from those of the three-point functions of scalar BMN operators by interchanging the coefficient for the symmetric-traceless representation with the coefficient for the singlet. We conclude that the $\mathbb{Z}_2$ symmetry of the pp-wave string theory is not manifest at the level of field theory three-point correlators.

Keywords: Duality in Gauge Field Theories, Penrose limit and pp-wave background, Supersymmetry and Duality.

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1. Introduction

The pp-wave/SYM correspondence of Berenstein, Maldacena and Nastase (BMN) [1] represents all massive modes of type-IIB superstring theory in a plane wave background in terms of composite BMN operators in $\mathcal{N} = 4$ Super Yang-Mills in four dimensions. Until now, most of the calculations on the gauge theory side of the correspondence were restricted to the BMN operators with scalar impurities.

The goal of the present paper is to extend the study of correlation functions of scalar BMN operators [2]–[8] to correlators of vector BMN operators. In particular we will address the relevance of a $\mathbb{Z}_2$ symmetry of the pp-wave string theory for the three-point functions of vector BMN operators in the gauge theory. Two-point correlators of BMN operators with vector impurities have already been considered in [9, 10, 11]. We will compute three-point functions of BMN operators with two vector impurities. These three-point functions are essential for the vertex-correlator pp-wave duality [2, 3, 7]. Our goal is to compare the coefficients of the three-point functions of vector BMN operators with those for scalar BMN operators. One would expect that the $\mathbb{Z}_2$ symmetry of string theory in the pp-wave background (explained below) requires the equality of these two coefficients. The main result of this paper is that this two coefficients are different.[1] Our result is that the vector three-point function (3.25) is related to the scalar three-point function (3.6) by exchanging the contribution for the symmetric-traceless operator with that of the singlet. This conclusion can also be derived from an earlier work of Beisert [10]. From these results it appears that the $\mathbb{Z}_2$ symmetry of the pp-wave string theory is not respected at the level of three-point functions of BMN operators with definite scaling dimensions in interacting field theory.

[1] The earlier version of this paper reported an agreement between the vector and scalar coefficients. This was due to an incorrect handling of the compensating terms in the vector BMN operators, last term in (2.4).

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On the string theory side, the pp-wave background has a bosonic symmetry of $SO(4)_1 \times SO(4)_2 \times \mathbb{Z}_2$, where the $\mathbb{Z}_2$ exchanges the action of the two SO(4) groups. This symmetry acts quite trivially at the free string level [12, 13]. However, its realisation in the dual field theory is not manifest and, therefore, highly non-trivial. In the pp-wave/SYM duality, the rotation groups $SO(4)_1 \times SO(4)_2$ in the lightcone-gauge string theory are mapped to the product of the Lorentz (euclidean) symmetry and the R-symmetry, $SO(4)_{\text{Lorentz}} \times SO(4)_R$, in the field theory. Thus, on the field theory side, the $\mathbb{Z}_2$ factor swaps the action of $SO(4)_{\text{Lorentz}}$ with $SO(4)_R$. A symmetry between spacetime and the internal (R-)space is novel, and might possibly be expected only in the large-N double scaling limit. The understanding of the $\mathbb{Z}_2$ symmetry, both in interacting string theory [12, 13] and in field theory, is one of the most challenging and exciting topics in the pp-wave/SYM duality.

In field theory, the BMN operators that are dual to string excitations in the first four directions, i.e. related to the factor $SO(4)_1$, carry impurities of the form $D_\mu Z$ (vector impurities). Two-point functions and anomalous dimensions of conformal primary vector BMN operators have been considered and determined in [9, 11]. The minimal form of the BMN correspondence is based on the mass-dimension type duality relation which maps the masses of string states to the anomalous dimensions of the corresponding BMN operators in the gauge theory:

$$H_{\text{string}} = H_{\text{SYM}} - J\,. \tag{1.1}$$

This relation has been verified for scalar BMN operators in the planar limit of SYM perturbation theory in [1, 14, 15]. Calculations in the BMN sector of gauge theory at the nonplanar level were performed in [16, 2, 4, 5] also taking into account mixing effects of planar BMN operators. The relation was extended in [18]–[21] to all orders in the effective genus expansion parameter g_2. In [9, 11] anomalous dimensions of vector BMN operators were found to be equal to those of scalar BMN operators. This verifies the consistency of the $\mathbb{Z}_2$ symmetry with the relation (1.1).

However, no further statement has been made so far about the $\mathbb{Z}_2$ symmetry beyond the mass-dimension duality (1.1). As we mentioned earlier, we find that the $\mathbb{Z}_2$ symmetry of the pp-wave string theory is not respected at the level of three-point functions of BMN operators with definite scaling dimensions in interacting field theory.

We will first need to carry out a field theory analysis of the three-point function involving BMN operators with vector impurities. This part of the analysis is new and contains some of the main results of this paper.

Let us recall that in a conformal theory, two- and three-point functions of conformal primary operators are completely determined by conformal invariance. One can always choose a basis of *scalar* conformal primary operators such that the two-point functions take the canonical form:

$$\langle \mathcal{O}_I(x_1)\mathcal{O}_J^\dagger(x_2)\rangle = \frac{\delta_{IJ}}{(x_{12}^2)^{\Delta_I}}\,, \tag{1.2}$$

and all the nontrivial information of the three-point function is contained in the x-independent coefficient C_{123}:

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3^\dagger(x_3)\rangle = \frac{C_{123}}{(x_{12}^2)^{\frac{\Delta_1+\Delta_2-\Delta_3}{2}}(x_{13}^2)^{\frac{\Delta_1+\Delta_3-\Delta_2}{2}}(x_{23}^2)^{\frac{\Delta_2+\Delta_3-\Delta_1}{2}}}\,, \tag{1.3}$$

where $x_{ij}^2 := (x_i - x_j)^2$. Since the form of the x-dependence of conformal three-point functions is universal, it is natural to expect that the spacetime independent coefficient C_{123} is related to the interaction of the corresponding three string states in the pp-wave background. Note that, in order to be able to use the coefficients C_{123}, it is essential to work on the SYM side with Δ-BMN operators. These operators are defined in such a way that they do not mix with each other (i.e. have definite scaling dimensions Δ) and are conformal primary operators. Conformal invariance of the $\mathcal{N}=4$ theory then implies that the two-point correlators of scalar Δ-BMN operators are canonically normalized, and the three-point functions take the simple form (1.3). Defined in this way, the basis of Δ-BMN operators is unique and distinct from other BMN bases considered in the literature. For two scalar impurities, this Δ-BMN basis was constructed in [4].

However, due to their nontrivial transformation properties under the conformal group, conformal primary *vector* BMN operators have in general more complicated two- and three-point functions. Thus, a priori, it is not clear whether it is possible (and how) to extract in the vector case a spacetime independent coefficient, similar to the C_{123} of the scalar correlators, that can then be compared with the pp-wave string interaction. In our opinion, this is one of the main obstacles in the understanding of how the pp-wave/SYM duality works for vector impurities and of the rôle of the $\mathbb{Z}_2$ symmetry beyond the level of the two-point functions in the pp-wave/SYM correspondence. In this paper, we make the observation that in a certain large distance limit, the two- and three-point correlation functions for vector BMN operators reduce to the same form as that for the scalar case. This allows one to make a direct comparison with the corresponding scalar three-point functions.

The paper is organised as follows. In section 2, we present the BMN operators with vector impurities and with positive R-charge. To obtain non-vanishing correlators, one also needs to know the conjugate BMN operators, i.e. the BMN operators with negative R-charge. We construct these operators by employing a new conjugation operation which is a product of the usual hermitian conjugation with the inversion operation. We explain why this construction is the most natural one in the present context. An important advantage of our construction is that the vector BMN operators are orthonormal with respect to the inner product defined using this conjugation. In section 3 we compute the three-point functions involving vector-BMN operators with definite scaling dimensions in interacting field theory.

Note on notation and conventions. We write the bosonic part of the $\mathcal{N}=4$ lagrangian as

$$\mathcal{L} = \frac{2}{g^2}\,\mathrm{Tr}\left(\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2}(D_\mu\varphi_i)(D_\mu\varphi_i) - \frac{1}{4}[\varphi_i,\varphi_j][\varphi_i,\varphi_j]\right)\,, \tag{1.4}$$

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where φ_i, $i = 1, \ldots, 6$ are the six real scalar fields transforming under an R-symmetry group SO(6). The covariant derivative is $D_\mu \varphi_i = \partial_\mu \varphi_i - i[A_\mu, \varphi_i]$, where $A_\mu = A_\mu^a T^a$, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$. If we define the complex combinations

$$\phi^1 = \phi = \frac{\varphi_1 + i\varphi_2}{\sqrt{2}}, \qquad \phi^2 = \psi = \frac{\varphi_3 + i\varphi_4}{\sqrt{2}}, \qquad \phi^3 = Z = \frac{\varphi_5 + i\varphi_6}{\sqrt{2}}, \tag{1.5}$$

the $\mathcal{N} = 4$ lagrangian can be re-expressed as

$$\mathcal{L} = \frac{2}{g^2} \operatorname{Tr} \left(\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \overline{(D_\mu \phi_I)} (D_\mu \phi^I) \right) + V_F + V_D \,, \tag{1.6}$$

where

$$\begin{aligned} V_F &= -\frac{2}{g^2} \operatorname{Tr} \left([\phi^I, \phi^J][\bar{\phi}_I, \bar{\phi}_J] \right) \;=\; -2\, \frac{2}{g^2} \operatorname{Tr} \left(Z\phi\bar{Z}\bar{\phi} - \phi\bar{\phi}\bar{Z}Z + \cdots \right), \\ V_D &= \frac{1}{2}\frac{2}{g^2} \operatorname{Tr} \left([\phi^I, \bar{\phi}_I][\phi^J, \bar{\phi}_J] \right) = \frac{2}{g^2} \operatorname{Tr} \left(Z\bar{Z}Z\bar{Z} - ZZ\bar{Z}\bar{Z} + \cdots \right) \end{aligned} \tag{1.7}$$

are the F-term and D-term of the scalar potential respectively. In the last equalities we write only the terms which will be relevant for our analysis. Our SU(N) generators are normalised as

$$\operatorname{Tr} \left(T^a T^b \right) = \delta^{ab} \,, \tag{1.8}$$

so that, for example,

$$\left\langle Z^i_j(x) \bar{Z}^l_m(0) \right\rangle = \frac{g^2}{2} \delta^i_m \delta^l_j \Delta(x) \,, \qquad \Delta(x) = \frac{1}{4\pi^2 x^2} \,. \tag{1.9}$$

The pp-wave/SYM duality is supposed to hold in the BMN large-N double scaling limit,

$$J \sim \sqrt{N} \,, \qquad N \to \infty \,. \tag{1.10}$$

In this limit there remain two free finite dimensionless parameters [1, 16, 2]: the effective coupling constant of the BMN sector of gauge theory,

$$\lambda' = \frac{g^2 N}{J^2} = \frac{1}{(\mu p^+ \alpha')^2} \tag{1.11}$$

and the effective genus counting parameter

$$g_2 := \frac{J^2}{N} = 4\pi g_s (\mu p^+ \alpha')^2 \,, \tag{1.12}$$

of Feynman diagrams. The right hand sides of (1.11), (1.12) express λ' and g_2 in terms of the pp-wave string theory parameters.

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2. Conformal primary vector BMN operators

Here we will study the BMN operators with vector impurities.[2] We will be concerned with the operators

$$\mathcal{O}^J_{\rm vac} = \frac{1}{\sqrt{JN_0^J}}\,{\rm Tr}\, Z^J\,, \tag{2.1}$$

and, for $\mu,\nu = 1,\ldots,4$,

$$\mathcal{O}^J_{\mu\nu,n} = \mathcal{C}\left(\sum_{l=0}^{J} e^{\frac{2\pi i n l}{J}}\,{\rm Tr}\left[(D_\mu Z)Z^l(D_\nu Z)Z^{J-l}\right] + {\rm Tr}\left[(D_\mu D_\nu Z)Z^{J+1}\right]\right) + \cdots\,, \tag{2.2}$$

where we defined

$$\mathcal{C} := \frac{1}{2\sqrt{JN_0^{J+2}}}\,, \qquad N_0 := \frac{g^2}{2}\,\frac{N}{4\pi^2}\,. \tag{2.3}$$

The normalisation of the operator $\mathcal{O}^J_{\rm vac}$ is such that its two-point function takes the canonical form (1.2) in the planar limit. As for the vector BMN operator $\mathcal{O}^J_{\mu\nu,n}$, it is normalized in such a way that eq. (2.23) below holds. We note that this choice of normalisation constant $\mathcal{C}$ is different from that[3] adopted in [11].

The first operator, $\mathcal{O}^J_{\rm vac}$, is a chiral (half-BPS) primary operator, and corresponds to the vacuum state of pp-wave string theory. For $n \neq 0$, the second operator, $\mathcal{O}^J_{\mu\nu,n}$ is a non-chiral vector conformal primary BMN operator, and corresponds to a string state $|\alpha_n^{\mu\dagger}\alpha_{-n}^{\nu\dagger}\rangle$. Here μ and ν are indices of bosonic excitations of the first SO(4) in the lightcone pp-wave string theory.[4] The operator $\mathcal{O}^J_{\mu\nu,n}$ has a definite scaling dimension, $\Delta_n = \Delta^{(0)} + \delta_n$, which implies that the single-trace expression on the right hand side of (2.2) must be accompanied with multi-trace corrections (and other mixing effects) at higher orders in g_2 [22, 4]. The dots on the right hand side of (2.2) indicate these corrections. These mixing terms are important in general, but in this paper we will show how to calculate correlation functions involving operators (2.2) without the need of knowing the precise analytical expressions for these mixing terms.

To be more precise, we should distinguish between symmetric-traceless, antisymmetric and singlet representations:

$$\mathcal{O}^J_{(\mu\nu),n} = \mathcal{C}\left(\sum_{l=0}^{J} e^{\frac{2\pi i n l}{J}}\,{\rm Tr}\left[(D_{(\mu} Z)Z^l(D_{\nu)} Z)Z^{J-l}\right] + {\rm Tr}\left[(D_{(\mu} D_{\nu)} Z)Z^{J+1}\right]\right) + \cdots\,, \tag{2.4}$$

$$\mathcal{O}^J_{[\mu\nu],n} = \mathcal{C}\left(\sum_{l=0}^{J} e^{\frac{2\pi i n l}{J}}\,{\rm Tr}\left[(D_{[\mu} Z)Z^l(D_{\nu]} Z)Z^{J-l}\right]\right) + \cdots\,, \tag{2.5}$$

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[2] CSC and VVK acknowledge an early collaboration with Michela Petrini, Rodolfo Russo and Alessandro Tanzini on the radial quantisation method and its applications to vector BMN operators discussed in section 2 of this paper.

[3] In particular we have the same normalisation constant for both cases $n = 0$ and $n \neq 0$. This is related to our prescription for the operator conjugation and the definition of the inner product. We will explain how this prescription is dictated by the pp-wave/SYM correspondence.

[4] We adopt the convention that BMN operators with vector (resp. scalar) impurities correspond to bosonic excitations of the first (resp. second) SO(4) in the lightcone pp-wave string theory.

$$\mathcal{O}_n^J = \mathcal{C}\left(\sum_{l=0}^{J} e^{\frac{2\pi i n l}{J}}\ \mathrm{Tr}\left[(D_\mu Z)Z^l(D_\mu Z)Z^{J-l}\right]\right) + \cdots , \tag{2.6}$$

where

$$\mathcal{O}_{(\mu\nu)} = \frac{1}{2}\left(\mathcal{O}_{\mu\nu} + \mathcal{O}_{\nu\mu}\right) - \frac{1}{4}\delta_{\mu\nu}\mathcal{O}_{\rho\rho}\,, \tag{2.7}$$

$$\mathcal{O}_{[\mu\nu]} = \frac{1}{2}\left(\mathcal{O}_{\mu\nu} - \mathcal{O}_{\nu\mu}\right). \tag{2.8}$$

Notice that the compensating term $\mathrm{Tr}\left[(D_{(\mu}D_{\nu)}Z)Z^{J+1}\right]$ is present only in the definition of the symmetric-traceless operator in (2.4) and not in the singlet (2.6). The precise form of the operators (2.4)–(2.6) is determined by acting with supersymmetry transformations on the scalar BMN operators in (2.27), and it was first obtained in [10, eqs. (B.10), (B.11) and (B.12)], which are valid also at finite J. Our operators (2.4), (2.5) and (2.6) follow in the large-J limit from those in [10]. Supersymmetry dictates that the single-trace bosonic operators in (2.5) and (2.6) must be accompanied by fermionic bilinears $\sim g$ and scalar bilinears $\sim g^2$ — see [10, appendix B] for the precise form of these terms. All these corrections, as well as the multi-trace corrections, will not be relevant for the calculation of three-point functions presented in the following sections, hence we will include them in the dots in (2.4)–(2.6) and discard them.

For $n = 0$, the operator $\mathcal{O}^J_{(\mu\nu),0}$ is a supergravity translational descendant of the vacuum:

$$\begin{aligned}\mathcal{O}^J_{(\mu\nu),0} &= \mathcal{C}\left(\sum_{l=0}^{J}\mathrm{Tr}\left[(D_{(\mu}Z)Z^l(D_{\nu)}Z)Z^{J-l}\right] + \mathrm{Tr}\left[(D_{(\mu}D_{\nu)}Z)Z^{J+1}\right]\right)\\ &= \frac{\partial_{(\mu}\partial_{\nu)}\ \mathrm{Tr}\, Z^{J+2}}{2J^{3/2}\sqrt{N_0^{J+2}}}\,.\end{aligned} \tag{2.9}$$

This operator is protected, hence its conformal dimension is given by the engineering dimension.

We now note that the operators $\mathcal{O}^J_{\mu\nu,n}$ are not orthogonal with respect to the scalar product $\langle \mathcal{O}^{J\dagger}_{\mu\nu,n}(x)\mathcal{O}^J_{\rho\sigma,m}(y)\rangle$, and therefore cannot correspond to the (orthonormal) basis of string states $|\alpha_n^{\mu\dagger}\alpha_{-n}^{\nu\dagger}\rangle$ (at least not directly). For example, one has [11] for the translational descendant defined in (2.9),

$$\langle \mathcal{O}^{J\dagger}_{(\mu\nu),0}(x)\mathcal{O}^J_{(\rho\sigma),0}(0)\rangle = \frac{4J^2}{(x^2)^{J+4}}\frac{x_{(\mu}x_{\nu)}x_{(\rho}x_{\sigma)}}{x^4}\,, \tag{2.10}$$

which is non-zero for $\mu,\nu \neq \rho,\sigma$. We also note that, in order to keep the right hand side of (2.10) finite as $J \to \infty$, an additional factor of J^{-1} would be required in the definition (2.9) of $\mathcal{O}^J_{\mu\nu,0}$ [11].

The right hand side of (2.10) has nothing to do with an orthonormality of the string states. We therefore introduce a different notion of conjugation, which will allow a direct correspondence to string (and supergravity) states defined as *hermitian conjugation* followed by an *inversion*:[5]

(i) We define the *barred-operator* as

$$\bar{\mathcal{O}}^J_{(\mu\nu),n}(x) := \mathcal{C}x^{2(\Delta-2)}\Bigg(\sum_{l=0}^{J} e^{\frac{2\pi i n l}{J}}\,\mathrm{Tr}\left[(J_{(\mu\alpha}\bar{D}_\alpha x^2\bar{Z})\bar{Z}^l(J_{\nu)\beta}\bar{D}_\beta x^2\bar{Z})\bar{Z}^{J-l}\right]+$$
$$+\,\mathrm{Tr}\left\{\left[(J_{(\mu\alpha}\bar{D}_\alpha)(x^2J_{\nu)\beta}\bar{D}_\beta)x^2\bar{Z}\right]\bar{Z}^{J+1}\right\}\Bigg), \qquad (2.11)$$

where $J_{\mu\nu}(x) = \delta_{\mu\nu} - 2x_\mu x_\nu/x^2$ is the usual inversion tensor, in terms of which the jacobian of the inversion $x'_\mu = x_\mu/x^2$ is expressed $\partial x'_\mu/\partial x_\nu = J_{\mu\nu}(x)/x^2$.

(ii) We introduce the inner product

$$\lim_{x\to\infty}\langle\bar{\mathcal{O}}_1(x)\mathcal{O}_2(0)\rangle \qquad (2.12)$$

and,

(iii) propose the correspondence between field theory and string theory inner products:

$$\lim_{x\to\infty}\langle\bar{\mathcal{O}}_1(x)\mathcal{O}_2(0)\rangle \leftrightarrow \langle\alpha_1|\alpha_2\rangle, \qquad (2.13)$$

where $|\alpha_\mathrm{i}\rangle$ is the string state that is in correspondence with the field theory operator $\mathcal{O}_\mathrm{i}$.

We remark that the introduction of the barred-operator is completely natural in the context of the radial quantisation of field theory [23], where hermitian conjugation is always accompanied by an inversion. Indeed, under inversion a scalar field $\mathcal{O}_\Delta(x)$ of conformal dimension Δ transforms as [24, 25]

$$\mathcal{O}_\Delta(x) \to \mathcal{O}'_\Delta(x') = x^{2\Delta}\mathcal{O}_\Delta(x)\,, \qquad x_\mu \to x'_\mu = \frac{x_\mu}{x^2}\,. \qquad (2.14)$$

Differentiating both sides of (2.14) with respect to x'_μ we obtain

$$\partial'_\mu\mathcal{O}'_\Delta(x') = x^2J_{\mu\nu}(x)\partial_\nu[x^{2\Delta}\mathcal{O}_\Delta(x)]\,. \qquad (2.15)$$

Combining the action of hermitian conjugation with an inversion, we get

$$\overline{\partial_\mu\mathcal{O}}_\Delta(x) = x^2J_{\mu\nu}(x)\partial_\nu\;[x^{2\Delta}\mathcal{O}^\dagger_\Delta(x)]\,, \qquad (2.16)$$

from which it follows[6] that

$$\bar{\mathcal{O}}^J_{(\mu\nu),n}(x) = \mathcal{C}\Bigg(\sum_{l=0}^{J} e^{\frac{2\pi i n l}{J}}\,\mathrm{Tr}\left[(x^2J_{(\mu\alpha}\bar{D}_\alpha\;x^2\bar{Z})(x^2\bar{Z})^l(x^2J_{\nu)\beta}\bar{D}_\beta\;x^2\bar{Z})(x^2\bar{Z})^{J-l}\right]+$$
$$+\,\mathrm{Tr}\left\{\left[(x^2J_{(\mu\alpha}\bar{D}_\alpha)(x^2J_{\nu)\beta}\bar{D}_\beta)x^2\bar{Z}\right](x^2\bar{Z})^{J+1}\right\}\Bigg), \qquad (2.17)$$

which is the free-theory expression for (2.11).

[5] We illustrate the following procedure for the symmetric-traceless operators (2.4). The extension to the antisymmetric and singlet representations is straightforward.

[6] A note on conventions: a bar applied to a composite operator $\mathcal{O}$ will always mean hermitian conjugation times an inversion as in (2.11). For ordinary fields we continue to use $\bar{Z} = Z^\dagger$.

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We note that the expression for the string operator (2.2) can be more compactly written as [9, 11]

$$\mathcal{O}^J_{\mu\nu,n} = \frac{\mathcal{C}}{J}\sum_{i,j=1}^{J+2} e^{\frac{2\pi i n(j-i)}{J+2}} D^{x_i}_\mu D^{x_j}_\nu \,\mathrm{Tr}\left[Z(x_1)\cdots Z(x_{J+2})\right]\Big|_{x_1=\cdots=x_{J+2}=x} . \tag{2.18}$$

The corresponding expression for the free barred-operator is given then by

$$\begin{aligned}\bar{\mathcal{O}}^J_{\mu\nu,n} = {} & \frac{\mathcal{C}}{J}\sum_{i,j=1}^{J+2} e^{\frac{2\pi i n(i-j)}{J+2}} (x^2 J_{\mu\alpha}D_\alpha)^{x_i}(x^2 J_{\nu\beta}D_\beta)^{x_j} \times \\ & \times \mathrm{Tr}\left[x_1^2 Z(x_1)\cdots x_{J+2}^2 Z(x_{J+2})\right]\Big|_{x_1=\cdots=x_{J+2}=x} .\end{aligned} \tag{2.19}$$

We now apply (2.11), or, equivalently (2.17), to the protected supergravity operator in (2.9)

$$\bar{\mathcal{O}}^J_{(\mu\nu),0} = \frac{(x^2 J_{(\mu\alpha}\partial_\alpha)(x^2 J_{\nu)\beta}\partial_\beta)\,\mathrm{Tr}(x^2\bar{Z})^{J+2}}{2J^{3/2}\sqrt{N_0^{J+2}}} , \tag{2.20}$$

and (2.10) is now replaced by the inner product

$$\begin{aligned}\langle \bar{\mathcal{O}}^J_{(\mu\nu),0}(x)\mathcal{O}^J_{(\rho\sigma),0}(0)\rangle &= (x^2 J_{(\mu\alpha}\partial^x_\alpha)(x^2 J_{\nu)\beta}\partial^x_\beta)\ \partial^0_{(\rho}\ \partial^0_{\sigma)}\ (x^2)^{J+2}\frac{\langle \mathrm{Tr}\,\bar{Z}^{J+2}(x)Z^{J+2}(0)\rangle}{4J^3 N_0^{J+2}} \\ &= \delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\nu\rho} - \frac{1}{2}\delta_{\mu\nu}\delta_{\rho\sigma} .\end{aligned} \tag{2.21}$$

Unlike (2.10), this expression is consistent with an operator–supergravity-state correspondence. This is the first consistency check of our proposal (2.11) and (2.13).

We now move on to consider string states, and compute in the free theory the two-point function $\langle \bar{\mathcal{O}}^J_{\mu\nu,n}(x)\mathcal{O}^J_{\rho\sigma,m}(0)\rangle$ in the limit $x\to\infty$. To this end, it is convenient to observe that the only terms which survive in this overlap are the ones where one derivative operator originating from the barred operator and one from the unbarred operator act on the same propagator, $(x^2 J_{\mu\alpha}\partial^x_\alpha)\partial^y_\rho\langle[x^2\bar{Z}(x)]Z(y)\rangle$. For these terms

$$(x^2 J_{\mu\alpha}\partial^x_\alpha)\partial^y_\rho \frac{x^2}{(x-y)^2}\bigg|_{y=0} = (x^2 J_{\mu\alpha}\partial^x_\alpha) x^2 \frac{2(x-y)_\rho}{(x-y)^4}\bigg|_{y=0} = 2\,\delta_{\mu\rho} , \tag{2.22}$$

where we have used that $\partial_\alpha(x_\rho/x^2) = J_{\alpha\rho}/x^2$, and $J_{\mu\alpha}J_{\alpha\rho} = \delta_{\mu\rho}$. Keeping this in mind, one easily computes in the limit $x\to\infty$,

$$\begin{aligned}\langle \bar{\mathcal{O}}^J_{\mu\nu,n}(x)\mathcal{O}^J_{\rho\sigma,m}(0)\rangle &= \left(\frac{\mathcal{C}}{J}\right)^2\cdot 4J^3\left(\frac{g^2}{2\cdot(4\pi^2)}\right)^{J+2} N^{J+2}(\delta_{m,n}\delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{m,-n}\delta_{\mu\sigma}\delta_{\nu\rho}) \\ &= \delta_{m,n}\delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{m,-n}\delta_{\mu\sigma}\delta_{\nu\rho} .\end{aligned} \tag{2.23}$$

This result is again consistent with our operator-string state correspondence (2.13). This is the second, nontrivial consistency check of our proposal (2.11) and (2.13). The normalisation chosen in (2.2) was designed to lead, on the right hand side of (2.23), to the product of Kronecker deltas with coefficient equal to 1.

A few general remarks are in order:

1. In distinction with eqs. (29a)–(29d) of [11], in our case (2.23), the overlap between supergravity and string states vanishes.

2. On general grounds, conformal invariance requires that the two-point function of vector conformal primary operators of scaling dimension Δ should have the form [24, 25]:

$$\langle \mathcal{O}^{J\dagger}_{\alpha\beta,n}(x)\mathcal{O}^{J}_{\rho\sigma,m}(0)\rangle = \text{const.}\frac{\delta_{m,n}J_{\alpha\rho}J_{\beta\sigma}+\delta_{m,-n}J_{\alpha\sigma}J_{\beta\rho}}{x^{2\Delta}}\,. \tag{2.24}$$

In our approach, we amputate the coordinate dependence on the right hand side of (2.24), and contract vector indices with (appropriate tensor products of) the inversion tensor J, thus directly computing

$$\lim_{x\to\infty} x^{2\Delta}J_{\mu\alpha}J_{\nu\beta}\langle \mathcal{O}^{J\dagger}_{\alpha\beta,n}(x)\mathcal{O}^{J}_{\rho\sigma,m}(0)\rangle = \delta_{m,n}\delta_{\mu\rho}\delta_{\nu\sigma}+\delta_{m,-n}\delta_{\mu\sigma}\delta_{\nu\rho}\,, \tag{2.25}$$

see our result (2.23). We take the limit $x\to\infty$ because x of the barred-operator is the inversion of x' and, in the radial quantisation formalism, states are obtained from operators at the point $x'=0$. The corresponding state in radial quantisation would be

$$\langle 0|(\partial^{'}_{\mu}O^{\dagger'}_{\Delta})(x'=0) = \lim_{x\to\infty}\langle 0|(x^2 J_{\mu\alpha}(x)\partial_\alpha)[x^{2\Delta}O^{\dagger}_{\Delta}(x)]\,, \tag{2.26}$$

which is precisely our definition. The two-point functions of vector operators are now correctly normalised, and take the canonical form. As a result, they are suited for a correspondence with the (orthonormal) string theory basis of states.

3. For the BMN operators with scalar impurities,

$$\mathcal{O}^{J}_{ij,n} = \mathcal{C}_{\text{scalar}}\left(\sum_{l=0}^{J} e^{\frac{2\pi i n l}{J}}\,\text{Tr}\left[\varphi_i Z^l \varphi_j Z^{J-l}\right] - \delta^{ij}\,\text{Tr}(\bar{Z}Z^{J+1})\right)+\cdots\,, \tag{2.27}$$

one can follow the same procedure as above, and define the barred-operators as $\bar{\mathcal{O}}^{J}_{ij,n}(x) = x^{2\Delta}\mathcal{O}^{J\dagger}_{ij,n}(x)$. Obviously, whether or not we introduce an inversion for the scalar fields is rather irrelevant: all the previous results for scalar Green functions are modified in a straightforward manner and the relation (2.13) is verified. However, as we have shown, this leads to important differences for vector operators.

4. It has been argued already in [9, 10] that the vector conformal primary BMN operators, i.e. Δ-BMN operators with various numbers of vector impurities are bosonic supersymmetry descendants of the scalar conformal primary BMN operators. This construction has been systematically carried out in [10]. Supersymmetry is important as it ensures that BMN operators with one vector and one scalar impurity [9] or two vector impurities [11] have exactly the same anomalous dimension as BMN operators with two scalar impurities, [9, 10], in agreement with string theory expectations.

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3. Three-point functions of vector conformal primary BMN operators

Conformal invariance constrains the expression of three-point functions of conformal primary operators. For the particular class of three-point functions $\langle \mathcal{O}^{J_1}_{\rho\sigma,n}(x_1)\mathcal{O}^{J_2}_{\rm vac}(x_2) \times \mathcal{O}^{J\dagger}_{\mu\nu,n}(x_3)\rangle$, involving vector conformal primary operators with $J = J_1 + J_2$, one has

$$\langle \mathcal{O}^{J_1}_{\rho\sigma,n}(x_1)\mathcal{O}^{J_2}_{\rm vac}(x_2)\mathcal{O}^{J\dagger}_{\mu\nu,m}(x_3)\rangle = \frac{F(\rho_n\sigma_{-n},\,{\rm vac}|\,\mu_m\nu_{-m};x_{13})}{(x_{12})^{\Delta_1+\Delta_2-\Delta_3}(x_{13})^{\Delta_1+\Delta_3-\Delta_2}(x_{23})^{\Delta_2+\Delta_3-\Delta_1}}\,, \tag{3.1}$$

where $x_{ij} = x_i - x_j$, Δ_i's are the scaling dimensions of $\mathcal{O}^{J_1}_{\rho\sigma,n}(x_1)$, $\mathcal{O}^{J_2}_{\rm vac}(x_2)$ and $\mathcal{O}^{J}_{\mu\nu,m}(x_3)$ respectively; and $F(\rho_n\sigma_{-n},\,{\rm vac}|\,\mu_m\nu_{-m};x_{13})$ is a dimensionless function of x_{13}. In the quantum theory, $\Delta_1 = J_1 + 2 + \delta_n$, $\Delta_2 = J_2$, $\Delta_3 = J + 2 + \delta_m$, where δ_m, δ_n are the anomalous dimensions of $\mathcal{O}^{J}_{\mu\nu,m}$, $\mathcal{O}^{J_1}_{\rho\sigma,n}$. Therefore

$$\begin{aligned}
\Delta_1 + \Delta_2 - \Delta_3 &= \delta_n - \delta_m\,,\\
\Delta_1 + \Delta_3 - \Delta_2 &= 2(J_1+2) + \delta_n + \delta_m\,,\\
\Delta_2 + \Delta_3 - \Delta_1 &= 2J_2 + \delta_m - \delta_n\,.
\end{aligned} \tag{3.2}$$

Notice that the anomalous dimensions for vector conformal primary operators with one vector and one scalar impurity [9] or with two vector impurities [11] are the same as for the original BMN operators with two scalar impurities [1].

Conformal invariance requires $F(\rho_n\sigma_{-n},\,{\rm vac}|\,\mu_m\nu_{-m};x_{13})$ to depend on the vector indices μ, ν, ρ, σ through appropriate tensorial products of the inversion tensor, $J(x_{13}) \otimes J(x_{13})$, thus it contains x-dependence[7] and cannot be compared directly to the coefficient C_{123} of the scalar three-point function (1.3), nor with a three-string interaction vertex. As in the previous section, we propose to consider instead the three-point functions involving the barred-operators and, moreover, to work in the limit[8] $x_3 \gg x_1, x_2$. Using our definition (2.17) for the barred-operator, we will therefore compute

$$\begin{aligned}
\langle \mathcal{O}^{J_1}_{\rho\sigma,n}(x_1)\mathcal{O}^{J_2}_{\rm vac}(x_2)\bar{\mathcal{O}}^{J}_{\mu\nu,m}(x_3)\rangle \longrightarrow (x_3)^{2\Delta_3}\langle \mathcal{O}^{J_1}_{\rho\sigma,n}(x_1)\mathcal{O}^{J_2}_{\rm vac}(x_2)\mathcal{O}^{J\dagger}_{\mu\nu,m}(x_3)\rangle &=\\
= \frac{C(\rho_n\sigma_{-n},\,{\rm vac}|\,\mu_m\nu_{-m})}{(x_{12})^{\delta_n-\delta_m}}&\,,
\end{aligned} \tag{3.3}$$

for $x_3 \to \infty$ (and x_1, x_2 finite), where $\mathcal{O}^{J_1}_{\rho\sigma,n}$ and $\bar{\mathcal{O}}^{J}_{\mu\nu,n}$ are given by (2.2) and (2.11). This is one of the key observation of this paper. Now $C(\rho_n\sigma_{-n},\,{\rm vac}|\,\mu_m\nu_{-m})$ can be compared directly to the scalar three-point function coefficient $C(k_nl_{-n},\,{\rm vac}|\,i_mj_{-m})$, defined below.

The three-point functions of BMN operators with scalar impurities (2.27) have the form

$$\left\langle \mathcal{O}^{J_1}_{kl,n}(x_1)\mathcal{O}^{J_2}_{\rm vac}(x_2)\mathcal{O}^{J\dagger}_{ij,m}(x_3)\right\rangle = \frac{C(k_nl_{-n},\,{\rm vac}|\,i_mj_{-m})}{(x_{12})^{\Delta_1+\Delta_2-\Delta_3}(x_{13})^{\Delta_1+\Delta_3-\Delta_2}(x_{23})^{\Delta_2+\Delta_3-\Delta_1}}\,, \tag{3.4}$$

or, introducing the barred-operators and working in the limit $x_3 \to \infty$ (and x_1, x_2 finite),

$$\left\langle \mathcal{O}^{J_1}_{kl,n}(x_1)\mathcal{O}^{J_2}_{\rm vac}(x_2)\bar{\mathcal{O}}^{J}_{ij,m}(x_3)\right\rangle = \frac{C(k_nl_{-n},\,{\rm vac}|\,i_mj_{-m})}{(x_{12})^{\delta_n-\delta_m}}\,. \tag{3.5}$$

[7] See, e.g. , section III.2 of [26].

[8] As before, the limit is a consequence of the formalism of radial quantisation. We also note that translational invariance, broken by radial quantisation, is restored in this limit.

The expression for the coefficient of the three-point function for BMN operators with two scalar impurities is

$$C(k_n l_{-n},\, \mathrm{vac}|\, i_m j_{-m}) = C_{123}^{\mathrm{vac}} \frac{2\,\sin^2(\pi m y)}{y\,\pi^2(m^2-n^2/y^2)^2}\left(\delta_{i(k}\delta_{l)j}\; m^2 + \delta_{i[k}\delta_{l]j}\,\frac{mn}{y} + \frac{1}{4}\delta_{ij}\delta_{kl}\,\frac{n^2}{y^2}\right), \tag{3.6}$$

where $y = J_1/J$ is the R-charge ratio, $C_{123}^{\mathrm{vac}} = \sqrt{JJ_1J_2}/N$ and the symmetric traceless and antisymmetric traceless combinations of two Kronecker deltas are defined as

$$\delta_{i(k}\delta_{l)j} = \frac{1}{2}(\delta_{ik}\delta_{lj} + \delta_{il}\delta_{kj}) - \frac{1}{4}\delta_{ij}\delta_{kl}\,, \qquad \delta_{i[k}\delta_{l]j} = \frac{1}{2}(\delta_{ik}\delta_{lj} - \delta_{il}\delta_{kj})\,. \tag{3.7}$$

These results were first obtained in the simple case $n = 0$ in [3]. The general expression (3.6) was derived in [4].

We now explain how the computation of the vector three-point functions proceeds. In subsection 3.1 we will describe the free-theory computation, and devote 3.2 to the planar corrections at one-loop. In order to efficiently organise our analysis, we will make a step-by-step comparison with the known computation for the case of scalar impurities. More precisely, our strategy will consist in identifying the “building blocks” which lead to the expression (3.6) for the coefficient $C(k_n l_{-n},\, \mathrm{vac}|\, i_m j_{-m})$ of the three-point function of scalar BMN operators, and comparing them to the corresponding building blocks for the case of BMN operators with vector impurities.

3.1 The calculation in free theory

Let us briefly review the free theory computation for the three-point function with (complex) scalar impurities,[9] say ϕ and ψ [2]. For calculations with scalars we use the complex basis (1.5), but continue calling the BMN operators as $\bar{\mathcal{O}}^J_{ij,m}$ and $\mathcal{O}^{J_1}_{kl,n}$.

Obviously, to get a nonzero result an impurity in the barred operator $\bar{\mathcal{O}}^J_{ij,m}(x_3)$ must be contracted with an impurity in $\mathcal{O}^{J_1}_{kl,n}(x_1)$ and the result boils down to the evaluation of the Feynman diagram in figure 2, which gives

$$\text{free-scalar:} \quad \left(\frac{g^2}{2}\right)^2 \frac{1}{(4\pi^2 x_{31}^2)^2} P_{\mathrm{free}}\,. \tag{3.8}$$

The factor P_{free} comes from carefully summing the BMN phase factors over all the position of ϕ and ψ impurities in the operators. Its explicit form is given in appendix B, and will not be needed here. When $n = 0$, (3.8) is the only contribution to the three-point function at the free level. When $n \neq 0$ the mixing with multi-trace operators must be taken into account [4, 22] and will modify even free theory results at leading order in g_2. These mixing effects being added to the contributions of figure 2 lead to the result of (3.6) [4].

We now consider the vector impurity case. First, notice that, in the free theory, covariant derivatives can be replaced with simple derivatives. The second key observation is that, in the limit we are considering ($x_3 \to \infty$ and x_1, x_2 finite), the only nonvanishing

[9]For the considerations in free theory presented in this section, we can set all anomalous dimensions equal to zero.

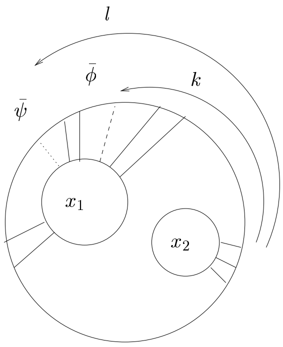

Figure 1: Three-point function with vector impurities. Free diagrams contributing to P_{free}.

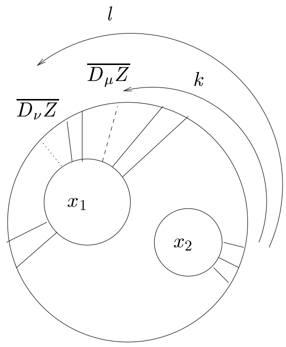

Figure 2: Three-point function with scalar impurities. Free diagrams contributing to P_{free}. The labels k and l count the Z-lines as indicated (for the diagram drawn above, $k = 2$, $l = 4$).

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contractions are those where an impurity in the operator $\bar{\mathcal{O}}^J_{\mu\nu,m}(x_3)$ is connected to an impurity in $\mathcal{O}^{J_1}_{\rho\sigma,n}(x_1)$. The result of such contractions has been analysed in (2.22). This observation leads to the immediate conclusion that there is only one Feynman diagram contributing to the free vector impurity case (figure 1). The associated phase factor is the same as for the scalar impurity case of figure 2. Therefore the free theory result for the vector three-point function is given by

$$\text{free-vector:} \qquad \left(\frac{g^2}{2}\right)^2 \frac{1}{(4\pi^2 x_{31}^2)^2} P_{\text{free}} \,. \tag{3.9}$$

In writing (3.9) we have taken into account that a factor of $2 \cdot 2$ from two free contractions of the vector impurities (see the right hand side of (2.22)) is precisely cancelled by a factor of $(1/2) \cdot (1/2)$ from the normalisation of the vector BMN operators.[10] Therefore the free result (3.9) for vector BMN operators leads to the same result as for the scalars (see (3.8)).

As in the scalar case, there are mixing effects of the barred single-trace operator with barred double-trace operators. These mixing effects will affect the free-theory contribution of figure 1. However, as we argued above, in the region $x_3 \gg x_1, x_2$, the vector impurities inside a BMN operator are orthonormal to each other with respect to the inner product (2.12), and hence behave in the same way as scalar impurities inside a BMN operator. As a result, it is easy to convince oneself that the modifications due to mixing effects to the free-theory three-point function coefficient are the same for both the scalar and the vector case. Hence, the free three-point function with vector impurities reproduces precisely its counterpart for the case of scalar impurities.

Before concluding this section, we would like to discuss further the issue of mixing. The mixing of single-trace BMN operators with double-trace operators is crucial in order to obtain conformal expressions such as (3.1) (or (3.3)). However, here we are not concerned with deriving the conformal expression on the right hand side of (3.1), which must be

[10] Notice that the normalisation constant $\mathcal{C}$ for vector BMN operators is half the normalisation of the scalars, $\mathcal{C} = (1/2) \cdot \mathcal{C}_{\text{scalar}}$.

correct anyway, as far as the mixing effects are such that we are dealing with vector conformal primary operators. Our goal is rather to compute the coefficient of the three-point function with two non-chiral operators, $C(\rho_n\sigma_{-n},\ \mathrm{vac}|\,\mu_m\nu_{-m})$. At leading order in g_2, the only mixing effect which contributes to the right hand side of (3.1) (or (3.3)) is the mixing of the barred operator with double-trace operators[11] [4]. These mixing effects will affect not only the free-theory contribution to $C^{\mathrm{free}}(\rho_n\sigma_{-n},\ \mathrm{vac}|\,\mu_m\nu_{-m})$, but also the logarithmic terms $\lambda' \log x_{13}^2$ and $\lambda' \log x_{23}^2$ due to interactions of the double-trace corrections in $\bar{\mathcal{O}}^J_{\mu\nu,m}(x_3)$ with the BMN operators sitting at x_1 and x_2. However, it is important to note that these mixing effects cannot affect the remaining logarithm, $\lambda' \log x_{12}^2$ [8]. Hence the coefficient of this logarithm can be computed in planar perturbation theory at order λ' without taking into account mixing altogether.

Our programme will therefore consist in assuming the conformal form (rather than deriving it), and evaluating the terms proportional to $\lambda' \log x_{12}^2$, thus determining the full coefficient of the vector three-point function. In doing so we are allowed to neglect the double-trace corrections, and work directly with the original single-trace BMN expressions.

3.2 The calculation in the interacting theory

The observations made at the end of the last section allow us to limit ourselves to the Feynman diagrams which can generate a $\log x_{12}^2$ term. Notice that self-energy corrections cannot generate such a $\log x_{12}^2$ dependence, and will thus be completely irrelevant for our purposes.

To begin with, let us recall the situation in the case of scalar impurities. In that case there are two diagrams contributing to this process, see figure 3. They come from an F-term in the lagrangian, $-V_F = 2\cdot 2/g^2\ \mathrm{Tr}\left(Z\phi\bar{Z}\bar{\phi} - \phi\bar{\phi}\bar{Z}Z\right) + \cdots$. In the first diagram (type I) the impurity goes across, and the diagram comes with coefficient $2\cdot 2/g^2$. In the second diagram the impurity goes straight (type II), and the diagram has a coefficient $-2\cdot 2/g^2$.

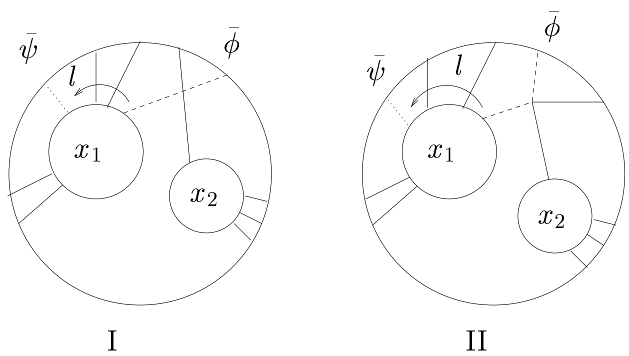

Figure 3: Interacting diagrams. Type I: impurity goes across. Type II: impurity goes straight.

[11] To see it immediately, note that the double-trace corrections to the single-trace expression for a BMN operator is of $\mathcal{O}(g_2)$, i.e. suppressed with $1/N$. This can be compensated by factorising the three-point function into a product of two two-point functions. This is possible only for the double-trace mixing in the operator $\bar{\mathcal{O}}$.

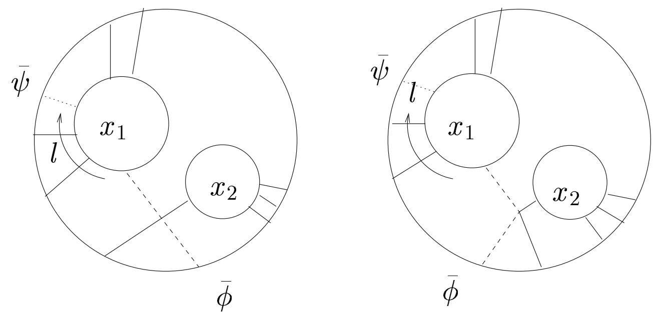

Figure 4: Interacting "mirror" diagrams.

The terms proportional to the $\log x_{12}^2$ resulting from these two diagrams are given by[12]

$$\text{type I-scalars:}\quad +2\left(\frac{g^2}{2}\right)^3 P_I X\,, \tag{3.10}$$

$$\text{type II-scalars:}\quad -2\left(\frac{g^2}{2}\right)^3 P_{II} X\,, \tag{3.11}$$

where the function X is

$$X = -\frac{1}{2^8\pi^6 x_{31}^4}\log x_{12}^2 = \frac{\log|x_{12}|^{-1}}{8\pi^2}\,[\Delta(x_{13})]^2\,, \tag{3.12}$$

see (A.1) and (A.2) of appendix A for further details. The overall factor $(g^2/2)^3$ comes from the insertion of one vertex $(2/g^2)$, and four propagators, $(g^2/2)^4$. The factors of $1/4\pi^2$ coming from the propagators are already included in the definition of X. Finally, P_I and P_{II} are the factors associated with the diagrams of type I and II, respectively. Their expressions are given in appendix B.

The diagrams drawn in figure 3 are also accompanied by "mirror" diagrams, where the interaction occurs in the bottom part of the external circle (which represents the barred trace operator) instead of in the upper part. These diagrams are represented in figure 4. Their effect is to add to the phase factors P_I and P_{II} their complex conjugates, $\bar P_I$ and $\bar P_{II}$. Finally, there are also the diagrams where the interaction involves the impurity ψ instead of ϕ. The net effect of these diagrams is to double up each phase factor, so that and amounts to replacing P_I and P_{II} in (3.10) and (3.11) respectively by $2(P_I+\bar P_I)$ and $2(P_{II}+\bar P_{II})$. Notice that (3.10) and (3.11) must be compared to the free result, which was computed in (3.8).

We have now assembled the building blocks eqs. (3.10), (3.11) for deriving the formula (3.6) for the case of different scalar impurities ($i\neq j$, $k\neq l$). In fact, the derivation of (3.6) follows immediately by tracking the $\log x_{12}^2$ terms. For the case of same impurities, one cannot use the complex basis in order to derive (3.6), but a straightforward modification of the above shows that (3.6) holds.

[12] To keep the formulae as simple as possible, we write down only multiplicative factors of $g^2/2$ and $1/(4\pi^2)$ coming from the vertices and the propagators involved in the interaction.

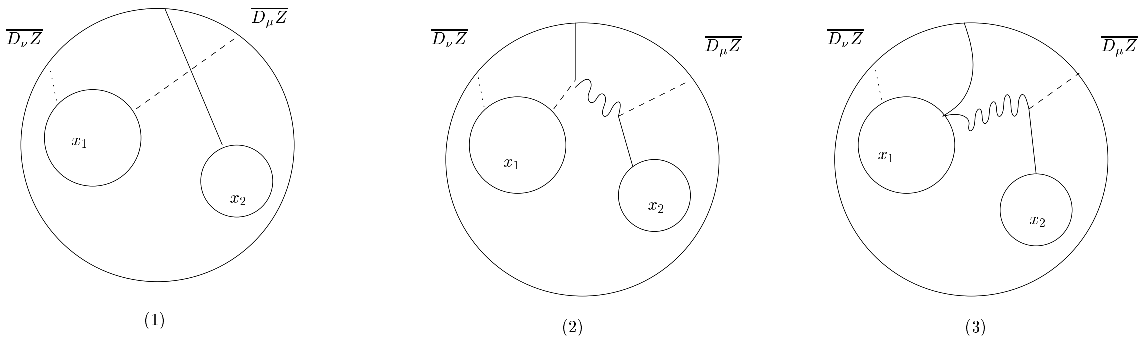

Figure 5: Interacting vector diagrams: type I.

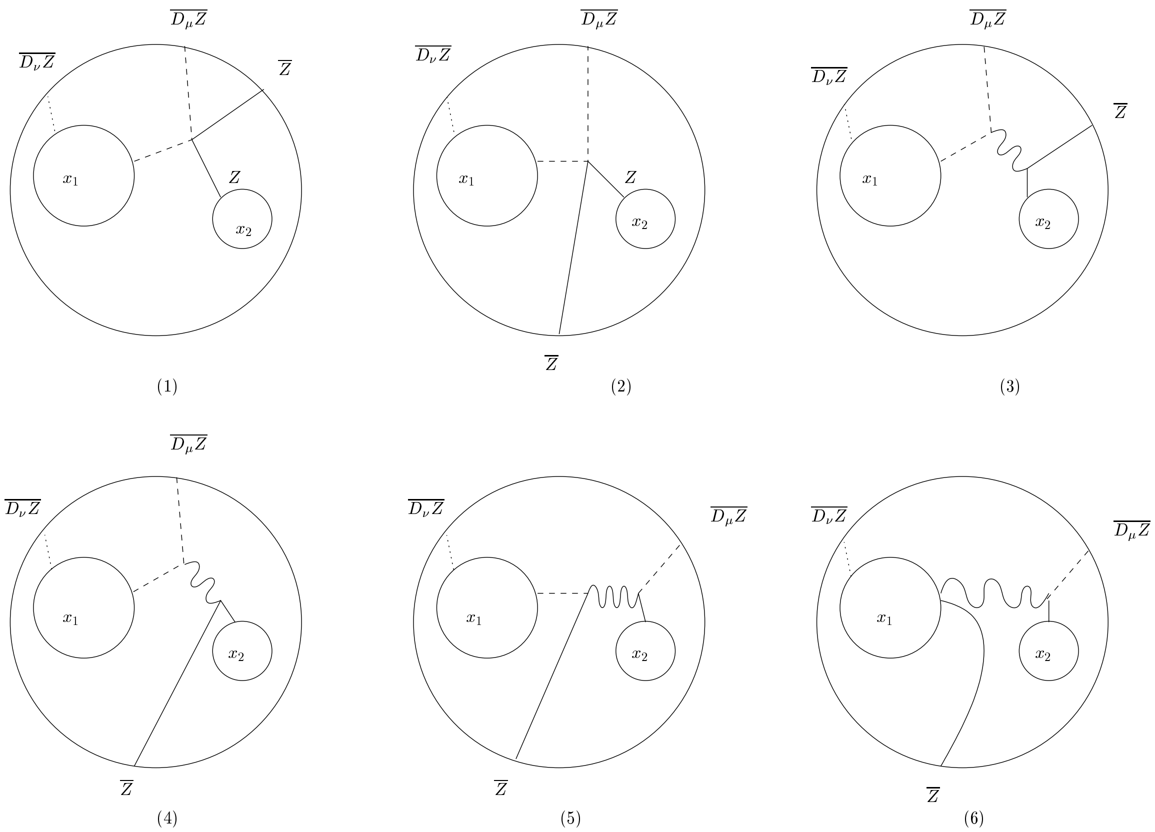

Figure 6: Interacting vector diagrams: type II.

Having identified the building blocks for the scalar-impurities calculation, we are finally ready to study the vector-impurities interacting case.

The diagrams where the interaction does not include either of the impurities cancel among each other in both the scalar and the vector cases. We are thus left with only two classes of diagrams, in complete analogy with the scalar case: in the diagrams of the first class, represented in figure 5, the impurity goes across (type I), whereas for those in the second class, in figure 6, the impurity goes straight (type II). From these diagrams it follows immediately that the phase factor associated with type-I (II) vector diagrams is the same as for the corresponding diagrams of type I (II) for scalars. To establish the $\mathbb{Z}_2$ symmetry we only need to compare the coefficients of the scalar and vector diagrams.

Let us have a closer look at the diagrams of type I (impurity goes across). The first diagram in figure 5 comes from a D-term in the lagrangian, $-V_D = 2/g^2\,\mathrm{Tr}\left(ZZ\bar{Z}\bar{Z}\right)+\cdots$. Importantly, it has the same sign of the F-term contributing to the same class of diagram for the scalar case, $-V_F = 2\cdot 2/g^2\,\mathrm{Tr}\left(Z\phi\bar{Z}\bar{\phi}\right)+\cdots$. Its contribution is

$$+\left(\frac{2}{g^2}\right)\left(\frac{g^2}{2}\right)^4 P_I X\,. \tag{3.13}$$

The second diagram in figure 5 is evaluated in (A.16) appendix A and gives a vanishing contribution. The third diagram is the gluon emission from the impurity, and comes from the commutator term in the covariant derivative impurity in $\mathcal{O}^{J_1}_{\rho\sigma,n}(x_1)$. This diagram is also evaluated in (A.11) of appendix A, and the result is:

$$+\,3\left(\frac{2}{g^2}\right)^2\left(\frac{g^2}{2}\right)^5 P_I X\,. \tag{3.14}$$

The total answer for the diagrams of type I is therefore equal to

$$\text{type I-vector:}\quad +4\cdot 2\cdot\frac{1}{4}\left(\frac{g^2}{2}\right)^3 P_I X\,. \tag{3.15}$$

As in the scalar case, the inclusion of mirror diagrams and of the diagrams where the interaction occurs where the other impurity is located, amounts to replacing P_I in (3.15) with $2(P_I+\bar{P}_I)$.

In writing (3.15) we have multiplied the sum of (3.13) and (3.14) by a factor of 2 from the free contraction of the impurity which does not interact, and a factor of 1/4 from the normalisations of the vector BMN operators. As in the scalar case, the inclusion of mirror diagrams and of the diagrams where the interaction occurs where the other impurity is located, amounts to replacing P_I in (3.15) with $2(P_I+\bar{P}_I)$. In conclusion, the coefficient of the $\log x_{12}^2$ term (3.15) arising from the diagrams of type I precisely coincides with the corresponding coefficient for type I diagrams for the scalar three-point functions, (3.10).

We now consider the diagrams of type II (figure 6). The first and second one originate from the two terms contained in $-V_D = 2/g^2\,\mathrm{Tr}\left(ZZ\bar{Z}\bar{Z}-Z\bar{Z}Z\bar{Z}\right)+\cdots$ respectively, and have opposite signs. The second diagram carries a symmetry factor 2 compared to the first and their spatial dependence is the same. Their combined result is equal to

$$(1-2)\left(\frac{2}{g^2}\right)\left(\frac{g^2}{2}\right)^4 P_{II} X\,. \tag{3.16}$$

The third and fourth diagram come with opposite signs and have the same spatial dependence, therefore their net contribution vanishes. The fifth diagram vanishes by itself, as the second diagram in figure 5. The sixth diagram follows from a contribution from the commutator term in the covariant-derivative impurity present in $\mathcal{O}(x_1)$. The sign of this diagram is opposite to to the similar one of type I (the third in figure 5), however this time it comes with phase factor P_{II}. Its contribution is therefore equal to

$$-\,3\left(\frac{2}{g^2}\right)^2\left(\frac{g^2}{2}\right)^5 P_{II} X\,. \tag{3.17}$$

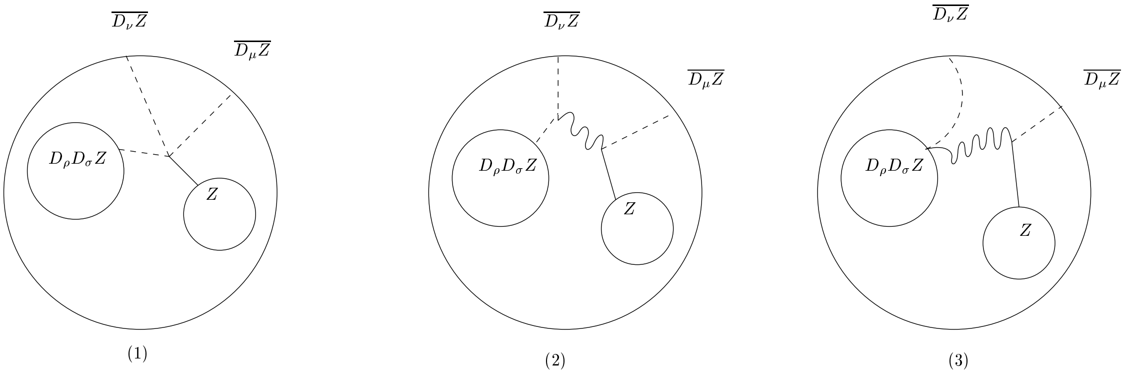

Figure 7: Diagrams from the compensating term, with phase factor 1.

The final result for the diagrams of type II is

$$\text{type II-vector:} \quad -4 \cdot 2 \cdot \frac{1}{4}\left(\frac{g^2}{2}\right)^3 P_{II} X\,. \tag{3.18}$$

As before, in writing the result (3.18) we have multiplied the sum of (3.16) and (3.17) by a factor of 2 from the free contraction of the non-interacting impurity, and a factor of 1/4 from the normalisations of the two vector BMN operators. Including mirror diagrams and the diagrams where the interaction occurs where the other impurities is located amounts to replacing P_{II} in (3.18) with $2(P_{II} + \bar{P}_{II})$. Therefore, we conclude that the coefficient of the $\log x_{12}^2$ term (3.18) from the diagrams of type II precisely matches the corresponding coefficient from type II diagrams for the scalar three-point functions, (3.11).

For the vector BMN operators in the antisymmetric and in the singlet representations, where the compensating term $\mathrm{Tr}\left[(D_{(\mu}D_{\nu)}Z)Z^{J+1}\right]$ is not present, the diagrams of type I and II give the full answer. Defining

$$P_{m,n} := 2(P_I + \bar{P}_I - P_{II} - \bar{P}_{II}) = -\frac{8m}{m - \frac{n}{y}} \sin^2 \pi m y\,, \tag{3.19}$$

the coefficients of the three-point function for the antisymmetric and singlet representations are respectively expressed in terms of the combinations

$$P_{m,n} - P_{-m,n} = -8\sin^2 \pi m y \frac{2\, mn/y}{m^2 - \frac{n^2}{y^2}}\,, \tag{3.20}$$

$$P_{m,n} + P_{-m,n} = -8\sin^2 \pi m y \frac{2m^2}{m^2 - \frac{n^2}{y^2}}\,. \tag{3.21}$$

For vector BMN operators in the symmetric-traceless representation, however, the contribution coming from the compensating term in (2.4) are important and must be included. In figure 7 we draw the corresponding Feynman diagrams which are associated with a phase factor equal to 1.

The sum of the first and the second diagram (gluon interaction) in figure 7 gives a contribution

$$2 \cdot \frac{1}{4}\left(\frac{2}{g^2}\right)^2 \left(\frac{g^2}{2}\right)^5 X\,. \tag{3.22}$$

The third diagram in figure 7 is the gluon emission from the impurity, and gives a contribution

$$3 \cdot 2 \cdot \frac{1}{4} \left(\frac{2}{g^2} \right)^2 \left(\frac{g^2}{2} \right)^5 X \,. \tag{3.23}$$

In the expressions (3.22) and (3.23) we have included a factor of $1/4$ from the normalisations of the vector BMN operators. In addition to the diagrams in figure 7, there are diagrams which are obtained from them by pulling down the upper right leg. They come with a relative factor $-\bar{q}^{J_1+1}$, where $q = \exp 2\pi i m/J$, and the minus sign comes from pulling down the leg. We also have to include mirror diagrams, and the diagrams with μ and ν interchanged (which double up the result).

When $n = 0$, the operator at x_1 does not in fact contain gluons, as is clear from (2.9), hence the total gluon emission diagrams have to cancel for $n = 0$. This does happen since the gluon emission in the diagrams containing the compensating term precisely cancels[13] the sum of the contributions in (3.14) and (3.17) at $n = 0$.

The complete result for the symmetric-traceless representation is obtained by adding together the contributions arising from the diagrams of type I and II and all the contributions from the compensating term. The coefficients of the three-point function for the symmetric-traceless representation is therefore expressed in terms of

$$P_{m,n} + P_{-m,n} - (P_{m,0} + P_{-m,0}) = -8 \sin^2 \pi m y \frac{2n^2/y^2}{m^2 - \frac{n^2}{y^2}} \,. \tag{3.24}$$

Summarising, if we first ignore the compensating terms, the building blocks (3.15) and (3.18) for deriving the expression for the coefficient $C(\rho_n \sigma_{-n}, \mathrm{vac}|\, \mu_m \nu_{-m})$ of the vector three-point function are precisely the same as the building blocks (3.10), (3.11) for the scalar three-point function coefficient $C(k_n l_{-n}, \mathrm{vac}|\, i_m j_{-m})$ (which in turn lead to the expression (3.6)). However, in the vector case a compensating term is present in the symmetric-traceless representation, whereas in the scalar case the compensating term affects instead the singlet operator. Therefore, it follows that $C(\rho_n \sigma_{-n}, \mathrm{vac}|\, \mu_m \nu_{-m})$ is given by

$$\begin{aligned} C(\rho_n \sigma_{-n}, \mathrm{vac}|\mu_m \nu_{-m}) \;&=\; C_{123}^{\mathrm{vac}} \frac{2 \sin^2(\pi m y)}{y \pi^2 (m^2 - n^2/y^2)^2} \times \\ &\times \left(\delta_{\mu(\rho} \delta_{\sigma)\nu} \frac{n^2}{y^2} + \delta_{\mu[\rho} \delta_{\sigma]\nu} \frac{mn}{y} + \frac{1}{4} \delta_{\mu\nu} \delta_{\rho\sigma} m^2 \right) . \end{aligned} \tag{3.25}$$

This is one of the principal results of this paper. We note that (3.25) agrees with the expression proposed earlier in [10].[14] The vector three-point function (3.25) is related to the scalar three-point function (3.6) by simply interchanging the contribution for the

[13]Notice that the total phase factor for the diagrams containing the compensating term is, for large J, $2 \cdot \left(2 - q^{J_1} - \bar{q}^{J_1}\right) = 8 \sin^2 \pi m y = -P_{m,0}$.

[14]In the first version of this paper, contributions coming from the compensating term for the symmetric-traceless representation (the last term in the right-hand side of (2.4)) were overlooked, which resulted in an incorrect expression.

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symmetric-traceless with that of the singlet. Therefore, the $\mathbb{Z}_2$ symmetry of the pp-wave string theory is not respected at the level of three-point functions of the BMN operators with definite scaling dimension in interacting field theory.

In this paper, we have introduced a new notion of conjugation to define BMN operators with negative R-charge. The new conjugation is a composition of the usual hermitian conjugation with an inversion, and is entirely consistent with the spirit of radial quantization. Using this conjugation, we introduced a new inner product for the BMN operators, which is relevant for the pp-wave/SYM correspondence and maintains the orthonormality of the string states. We computed three-point functions for BMN operators with two vector impurities. The corresponding coefficient is given in (3.25). This expression should be contrasted with the result in the scalar case, (3.6). Contrary to expectations based on the $\mathbb{Z}_2$ invariance in pp-wave string theory, these two expressions are different.

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A. Evaluation of the diagrams

In the computation of the scalar three-point functions we define the function

$$X_{1234} = \int d^4z \Delta(x_1 - z)\Delta(x_2 - z)\Delta(x_3 - z)\Delta(x_4 - z)\,. \tag{A.1}$$

X_{1234} develops a $\log x_{12}^2$ term X as x_1 approaches x_2, which is given by

$$X_{1234}|_{x_3=x_4} = -\frac{1}{2^8\pi^6}\left(\frac{1}{x_{14}^2 x_{24}^2}\right)\log x_{12}^2\,. \tag{A.2}$$

One can also derive directly from (A.1) useful expressions for the derivatives of X,

$$\left(\frac{\partial}{\partial x_4^\alpha}X_{1234}\right)_{x_3=x_4} = \frac{1}{2}\left(-\frac{1}{2^8\pi^6}\right)\log x_{12}^2\frac{\partial}{\partial x_4^\alpha}\left(\frac{1}{x_{14}^2 x_{24}^2}\right), \tag{A.3}$$

$$\left(\frac{\partial^2}{\partial x_4^\alpha \partial x_4^\beta}X_{1234}\right)_{x_3=x_4} = \frac{1}{6}\left(-\frac{1}{2^8\pi^6}\right)\log x_{12}^2\frac{\partial^2}{\partial x_4^\alpha \partial x_4^\beta}\left(\frac{1}{x_{14}^2 x_{24}^2}\right), \tag{A.4}$$

where, on the right-hand side of (A.4) derivatives are taken as if $\alpha \neq \beta$, i.e. $\partial_\alpha x^\beta = 0$ rather than δ_α^β.

In the evaluation of all the diagrams with an insertion of quadrilinear term in the scalars coming from $-V_D$ we also made use of the following relations:

$$\left[J_{\mu\alpha}(x_3)\partial_\alpha^{x_3}x_3^2\partial_\rho^{x^1}X_{1234}\right]_{x_3=x_4} \longrightarrow \delta_{\mu\rho}X\,, \tag{A.5}$$

$$\left[J_{\mu\alpha}(x_4)\partial_\alpha^{x_4}x_4^2\partial_\rho^{x^1}X_{1234}\right]_{x_3=x_4} \longrightarrow \delta_{\mu\rho}X\,, \tag{A.6}$$

where equality with the right hand sides holds for the $\log x_{12}^2$ terms, in the limit $x_{12}\to 0$ and $x_3\to\infty$, and we used

$$\partial_\alpha\frac{x_\beta}{x^2} = \frac{J_{\alpha\beta}(x)}{x^2}\,. \tag{A.7}$$

The covariant derivative interaction term, which participates in the third diagram of figure 5 and in the sixth diagram of figure 6, is proportional to $(\partial_\mu^{x_2}-\partial_\mu^{x_3})Y_{123}$, where

$$Y_{123} = \int d^4z\Delta(x_1-z)\Delta(x_2-z)\Delta(x_3-z)\,. \tag{A.8}$$

It is easy to realise that, as $x_{12}\to 0$, the function Y_{123} contains a logarithmic term given by

$$Y_{123}|_{x_{12}\to 0} = -\frac{1}{2^6\pi^4}\log x_{12}^2\frac{1}{x_{13}^2}\,. \tag{A.9}$$

One also needs the following expression for the $\log x_{12}^2$ term in the first derivative of Y:

$$\left(\frac{\partial}{\partial x_1^\alpha}Y_{123}\right)_{x_{12}\to 0} = -\frac{1}{2^7\pi^4}\log x_{12}^2\frac{\partial}{\partial x_1^\alpha}\frac{1}{x_{13}^2}\,. \tag{A.10}$$

We note that (A.10) is obtained from (A.8) rather than by differentiating (A.9).

To compute the diagram, we also used that, as $x_{12}\to 0$,

$$[J_{\mu\beta}(x_3)\partial_\beta^{x_3}x_3^2](\partial_\rho^{x_2}-\partial_\rho^{x_3})Y_{123}\longrightarrow 3X\delta_{\mu\rho}\,. \tag{A.11}$$

The contribution of the second diagram in figure 5 (gluon interaction) is encoded in the function H defined by

$$H_{14,23} = (\partial_\mu^{x_1}-\partial_\mu^{x_4})(\partial_\mu^{x_2}-\partial_\mu^{x_3})\int d^4zd^4t\Delta(x_1-z)\Delta(x_4-z)\Delta(x_2-t)\Delta(x_3-t)\Delta(z-t)\,, \tag{A.12}$$

which can be evaluated with the useful relation proved in [4]

$$\frac{H_{14,23}}{\Delta_{14}\Delta_{23}} = X_{1234}\left(\frac{1}{\Delta_{12}\Delta_{43}}-\frac{1}{\Delta_{13}\Delta_{24}}\right)+G_{1,23}-G_{4,23}+G_{2,14}-G_{3,14}\,, \tag{A.13}$$

where $\Delta_{ij}=\Delta(x_i-x_j)$ and

$$G_{i,jk} = Y_{ijk}\left(\frac{1}{\Delta_{ik}}-\frac{1}{\Delta_{ij}}\right). \tag{A.14}$$

We can recast (A.13) as

$$\begin{aligned} H_{14,23} &= -X_{1234}\frac{\Delta_{14}\Delta_{23}}{\Delta_{13}\Delta_{24}}+\left(\frac{Y_{123}}{\Delta_{13}}+\frac{Y_{124}}{\Delta_{24}}\right)\Delta_{14}\Delta_{23}+\cdots \\ &= H_I+H_{II}+\cdots\,, \end{aligned} \tag{A.15}$$

where the dots stand for terms which either vanish or do not contain the $\log x_{12}^2$. The vanishing of the second diagram in figure 5 stems from the fact that

$$[J_{\mu\alpha}(x_3)\partial_\alpha^{x_3} x_3^2]\partial_\mu^{x_1} H_I = -[J_{\mu\alpha}(x_3)\partial_\alpha^{x_3} x_3^2]\partial_\mu^{x_1} H_{II}\,, \tag{A.16}$$

where the equality holds for the $\log x_{12}^2$ terms we are looking at, and in the limit $x_3 = x_4 \to \infty$.

B. Summing the BMN phase factors

We report here the expressions for the coefficients P_{free}, P_I and P_{II} which arise after summing over the BMN phase factors in the free-theory diagrams and in the diagrams of type I and II, respectively. Defining

$$q = e^{\frac{2\pi i m}{J}}\,, \qquad q_1 = e^{\frac{2\pi i n}{J_1}}\,, \tag{B.1}$$

we have, for the free case,

$$P_{\text{free}} = \sum_{k,l=0}^{J_1} (\bar{q}q_1)^{l-k} + \sum_{l=0}^{J_1} (\bar{q}q_1)^0 = \frac{J^2}{\pi^2}\frac{\sin^2 \pi m y}{(m-\frac{n}{y})^2} + \mathcal{O}(J)\,, \tag{B.2}$$

where the last equality holds in the BMN limit, and $y = J_1/J$. The expressions for P_I and P_{II} are given by

$$P_I = \sum_{l=0}^{J_1} (\bar{q}q_1)^l \bar{q}\,, \qquad P_{II} = \sum_{l=0}^{J_1} (\bar{q}q_1)^l\,. \tag{B.3}$$

The effective coefficient which multiplies the $\log x_{12}^2$ term in the three-point function, both in the scalar and in the vector case, is

$$\begin{aligned} 2(P_I + \bar{P}_I) - 2(P_{II} + \bar{P}_{II}) &= 2\sum_{l=0}^{J_1} (\bar{q}q_1)^l (\bar{q} - 1) + \text{c.c.} \\ &= -\frac{8m}{m - \frac{n}{y}} \sin^2 \pi m y\,. \end{aligned} \tag{B.4}$$

Again, the last equality holds in the BMN limit. Notice that (B.4) is of $\mathcal{O}(1/J^2)$ compared with P_{free} in (B.2), as it should. This, together with a factor g^2N of with the planar one-loop contribution to the three-point function, reconstruct the effective Yang-Mills coupling constant $\lambda' = g^2N/J^2$, which is kept fixed in the BMN limit.

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